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A PRELIMINARY DETERMINATION OF THE ABSORPTION OF LIGHT BY THE EARTH'S ATMOSPHERE.

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In almost all stellar photometric work, a knowledge of the amount of light absorbed by the Earth's atmosphere is necessary. In certain very refined variable star work, as accurate a determination as possible of this absorbing quality of the atmosphere is desirable.

It is known to all that if there were no atmosphere surrounding the Earth the stars would appear brighter. The air acts as an obscuring veil dimming their lustre.

The thicker the quantity of air through which the observer views the stars the greater will be their diminution in brightness.

The minimum diminution takes place when the star being observed is in the zenith; the maximum when it is on the horizon.

The amount of light lost by a star when in the zenith is called the *coefficient of atmospheric absorption.* It is the difference in brightness between what the star would be if there were no air, and what it is when viewed through about two hundred miles of air.

Once this coefficient is known, it is possible to determine the loss of light at any altitude, always assuming that the atmosphere is homogeneous in structure.

Many determinations of this coefficient have been made by astronomers at various parts and heights on the earth's surface.

These determinations naturally vary with the conditions under which they were made. It is matter of common experience that out on the Karoo the stars shine far more brilliantly than they do in the neighbourhood of Cape Town.

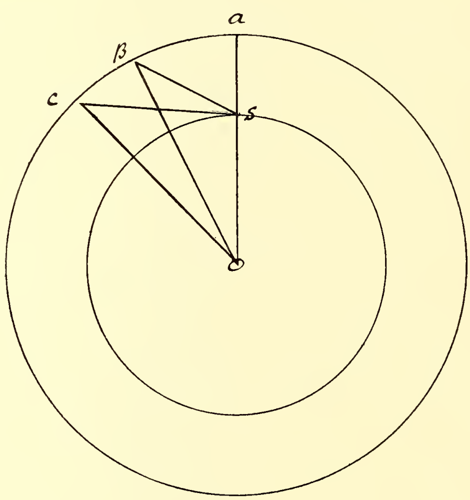
In this uncertainty it seemed to me desirable to make a determination of the coefficient of absorption for this part of Cape Colony—the Eastern province. This paper is a brief and preliminary statement of a single determination made on the summit of one of the hills of the Winterberg Range.

I shall divide my paper into three sections :—

(1) A statement of the Problem.
(2) Consideration of the Winterberg Observations.
(3) Deductions therefrom.

1. Statement of the Problem.

I shall only deal with simple relations, and thus present the problem in its simplest form.



Let—

s represent an observing station on the earth's surface,
sa, $s\beta$, sc different altitudes of a star when viewed from the station s,
sa the height of the atmosphere,
os the radius of the earth,

and let—

L = amount of light of a star seen in the zenith direction sa, *if there were no atmosphere*,
= m_0 in magnitudes,
Lq = amount of light of the same star seen in the direction sa,
= m, the apparent magnitude of the star,
Lq^t = amount of light of the same star seen in the direction $s\beta$,
= m_1.

Therefore, if K is the light ratio—

$$\frac{L}{Lq} = K^{m-m_0} \qquad \frac{Lq}{Lq^t} = K^{m_1-m}$$

$$\frac{L}{Lq} = K^{m_1-m_0}$$

$$\frac{1}{q^t} = K^{m_1-m_0}$$

or—

$$t \log\left(\frac{1}{q}\right) = (m_1 - m_0)\log K$$

$$m_1 - m_0 = t\left(\frac{\log\frac{1}{q}}{\log K}\right)$$

putting—

$$\frac{\log\frac{1}{q}}{\log K} = \gamma$$

$$m_1 - m_0 = t\gamma$$

now—

$$t = \frac{\beta s}{as}$$

let—

$$as\beta = \theta,$$
$$os = 1,$$
$$sa = d,$$

then it can be easily shown from a simple consideration of the figure that—

$$t^2d + 2t\cos\theta - d - 2 = 0,$$

or—

$$t = \frac{-\cos\theta + \sqrt{(d+1)^2 - \sin^2\theta}}{d},$$

and—

$$m_1 - m_0 = \frac{\gamma}{d}\left\{\sqrt{(d+1)^2 - \sin^2\theta} - \cos\theta\right\}\cdot$$

the right-hand number of this equation is the correction for atmospheric absorption, and includes as unknown quantities the height of the atmosphere $= d$, and the coefficient of absorption $= \gamma$.

This final equation can be put in the form—

$$(m_{\mathrm{I}}-m_{\mathrm{o}})^2+2(m_{\mathrm{I}}-m_{\mathrm{o}}) \cos \theta \frac{\gamma}{d}-\gamma^2\left(1+\frac{2}{d}\right) = 0.$$

Returning to the value of t, viz.—

$$t^2d+2t \cos \theta - d - 2 = 0,$$

we may remark that when d is very small, that is when the ratio of the height of the air to the radius of the earth is practically zero, the above expression becomes—

$$2t \cos \theta = 2,$$

$$t = \sec \theta,$$

and—

$$(m_{\mathrm{I}} - m_{\mathrm{o}}) = \gamma \sec \theta.$$

This limiting value holds good for zenith distances up to 70°, but after that it ceases to hold good.

The rigorous equation of condition—

$$(m_{\mathrm{I}} - m_{\mathrm{o}})^2+2(m_{\mathrm{I}} - m_{\mathrm{o}}) \cos \theta \frac{\gamma}{d}-\gamma^2\left(1+\frac{2}{d}\right) = 0,$$

well represents the relation between atmospheric absorption and zenith distance at any altitude, always premising that the atmosphere consists of homogeneous concentric spherical shells round the earth's surface, and that it is free of foreign bodies such as dust or masses of vapour.

The variations in the barometer for different regions of the earth's surface show that neither of these considerations is admissible.

2. Consideration of Winterberg Observations.

Over five hundred observations were made from the summit of a hill, whose altitude would be about 4,000 feet.

A number of stars were selected, and their decrease in brightness noted as they passed from the zenith to the horizon.

These quantities were plotted down and the interpolating curve drawn. The following are measures on the curve :—

Altitude 79°	Decrease = 0·64 m.
81	0·75
83	0·95
85	1·28
87	1·82
89	2·70

It was at first hoped that these observations would yield both the coefficient of absorption ($= \gamma$) and the height of the atmosphere ($= d$), but a variety of solutions confirmed me in the view that a far more refined series of observations would require to be made before any value of the height of the atmosphere worthy of credence would emerge from the equations.

In the present solution, therefore, the coefficient of absorption is expressed as a function of the height of the atmosphere.

We may, however, indicate an approximate solution in which the height of the atmosphere is also determined.

A first solution yields as a value of the coefficient of atmospheric absorption—

0·20 m.

As the magnitudes given above are those which are obtained from a comparison with zenith magnitudes, it is necessary to add to these differences the coefficient of absorption in order to obtain the absolute decrease in brightness due to the amount of atmosphere passed through. We may term these new quantities absolute values: they represent the difference between the observed brightness of a star at certain zenith distance and the real brightness of the star if there were no atmosphere.

The foregoing table of decrease in magnitude becomes therefore by the addition of the coefficient of absorption—

Zenith distance 79°	Decrease in brightness 0·84 m.
81	0·95
83	1·15
85	1·48
87	2·02
89	2·90

And the resulting equations of condition are—

$$(0{\cdot}84)^2 + 0{\cdot}321\,\frac{\gamma}{d} - \gamma^2\left(1 + \frac{2}{d}\right) = 0$$
$$(0{\cdot}95)^2 + 0{\cdot}296$$
$$(1{\cdot}15)^2 + 0{\cdot}281$$
$$(1{\cdot}48)^2 + 0{\cdot}257$$
$$(2{\cdot}02)^2 + 0{\cdot}210$$
$$(2{\cdot}90)^2 + 0{\cdot}099$$

From which we obtain as final equations—

$$6\gamma^2\left(1 + \frac{2}{d}\right) - 1{\cdot}464\,\frac{\gamma}{d} - 17{\cdot}611 = 0$$

$$+0{\cdot}390\,\frac{\gamma}{d} + \ 3{\cdot}117 = 0$$

a solution of which yields—

$$\gamma = 0{\cdot}17 \text{ m.}$$
$$d = 0{\cdot}005.$$

Reducing the coefficient of atmospheric absorption to sea-level, we have the final value for this important factor—

0·19 m.

The value of the height of the atmosphere obtained from the equations, viz., 0·005, or 20 miles, is of course too small. But this small value may mean that above 20 miles the air is too rare to stop light to any appreciable extent.

3. Deductions from the Above.

The principal determinations of the coefficient of absorption are—

Seidel	0·25 m.
Langley	0·13
Pritchard	0·19
Muller	0·21
Pickering	0·25

The mean of these results is—

0·21 m.,

agreeing very closely with that obtained at Lovedale—

0·19 m.

Taking the mean of all the results, we obtain as the value of the coefficient of atmospheric absorption—

0·20 m.,

which, interpreted into other terms, means that 17 per cent. of all rays that strike the atmosphere perpendicularly are absorbed by the atmosphere. That is, a star in the zenith shines with 83 per cent. of its intrinsic brightness; on the horizon this is reduced to such an extent that the star shines with only about one-fortieth of its zenith brightness.

The foregoing investigation was undertaken for the purpose of obtaining a value of the coefficient of absorption true for South Africa. The results obtained by different astronomers at different parts of the earth's surface indicates that it is not the same for all regions. The presence of moisture, of dust particles, will modify to an appreciable extent the absorbing quality of the atmosphere.

The present investigation is also, as the title indicates, simply preliminary. For a determination of the height of the atmosphere by this method the most refined photometric equipment is necessary. And I am confident that an investigation so carried out will yield results comparable in exactness and finality with any of the indirect ways of obtaining the height of the atmosphere.